Thermally activated breakdown in the fiber-bundle model

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Guarino *et al.*, (cond-Mat/9908329) have recently introduced a fiber bundle model where fiber fracture can be thermally activated. Under a fixed (subcritical) loading, the mean failure time of the bundle $\langle \tau_f \rangle$ is studied. An analytical expression for the latter is obtained as a function of the load. The effect of a (narrow) quenched disorder in the fracture stress of the fibers with a Gaussian distribution is shown to lead to an effective temperature simply translated with respect to the actual one. Finally, some ''critical'' properties of fracture precursors which have been proposed are investigated within the present model.

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I. INTRODUCTION

Damage mechanics describes the mechanical behavior of heterogeneous solids in which microcracks can nucleate, propagate and be arrested. Paradoxically, even though heterogeneity is essential for providing the basis of micro-crack nucleation and arrest, the constitutive law is generally considered as describing a homogeneous solid. This motivates studies of the effect of disorder on the mechanical behavior of quasibrittle materials. However, even minimal models in two or three dimensions represent a formidable challenge to solve analytically. Thus a lot of work has been based on numerical simulations. In this case, the strength of the conclusion which can be reached is severely limited by system size constraints and statistics. [1,2]

Alternatively, a lot of effort has been spent on analytical approaches which may provide a more solid ground. The class of models which can be addressed analytically also involve severe simplifications. One of these models, the ''fiber bundle'' model, received a lot of attention because one can solve analytically a number of its properties. This mean-field model has been introduced by Daniels [3] as early as in the 1940's. In the original version of this model, parallel fibers connected to two rigid bars are loaded in tension. All unbroken fibers are supposed to be subjected to an equal load (this is where the mean-field hypothesis comes into play). The behavior of each fiber is supposed to be ideally elastic (with the same stiffness for all fibers) and brittle (with a random distribution of failure strength). Randomness or disorder enters only in the definition of the breaking thresholds and is time independent. After this step, the time evolution of the bundle is deterministic. Many exact properties of this model have been obtained, from the mean force-displacement characteristics [3] to fine details on the statistics of avalanches. [4] A large variety of extensions have been considered, such as load sharing rules, [5–7] coupling to an elastic block, [8] plastic behavior [9] etc.

Among them an important class of extension has been focused on the statistics of time to failure, which concern materials where subcritical crack growth may take place. In this case, under a given (subcritical) loading one fiber may stand a given load for some time, until it breaks down. The question at the heart of these studies is to understand the statistics of the time to failure for the entire bundle. Coleman [10,11] pioneered such extensions, and obtained key results for the Gaussian character of the failure time distribution of short fibers (and the breakdown of such a character for long fibers). Later, some of these results have been extended to a broad class of time-dependent strength by Phoenix. [12,13]

Motivated by an initial theoretical work by Pomeau, [14] and by preceding experimental results [15,16] on the time to failure under a constant load, Guarino *et al.* [17] have introduced a variant of the latter class of extensions, taking into account a thermally activated fracture initiation. We stress the point that there are many different ways of introducing such thermally activated rupture. In particular, Phoenix and Tierney [18] derived a breakdown rule based on the interatomic potential between atoms as fitted by a Morse potential. The approach of Guarino is based on a different spirit, namely thermal fluctuation are assumed to induce an additional white Gaussian noise in the load carried by the fibers. Based on a numerical study of this model, they obtained results which were argued to support Pomeau's initial suggestion concerning the scaling of the time to failure, suitably adapted to this mean-field model. Moreover, a number of experimental results [15,16,20,21] seem also to conform to the scaling laws obtained within the model.

It is thus important to secure these results through an analytic investigation of the properties of this fiber bundle model. It is the purpose of the present article where we consider strictly the model introduced in Ref. [17]. Our analytical results support partly the results inferred from the numerical study. In particular the temperature dependence of the time to failure is justified (up to unimportant logarithmic correction) as well as the power-law distribution of energy dissipated in breaking events. However, some of the expressions proposed in Ref. [17] are unfounded, and an analytical solution is presented here.

Thus one of the principal motivations of the introduction of this model, i.e., a direct justification of Pomeau's conjecture, [14] has to be reconsidered carefully. We will not in this paper discuss the foundations of the model, but rather stick to its original definition. The results concern the mean value (and statistical distribution) of the failure time of a homogeneous bundle under a fixed load. We also consider the case of a heterogeneous fiber bundle and show that indeed the quenched disorder has (to first order) the effect of modifying the effective temperature, as suggested by Guarino *et al.* [17] Finally, we consider also the statistics of the energy released in the fiber failure as precursors to global failure.

II. HOMOGENEOUS SYSTEM

Let us consider first the case without any disorder in the fiber strength. The threshold force is $f_c = 1$. The total load on the bundle is F, and there are initially N fibers. The force on each fiber is

$$f = f_0 + \eta, \tag{1}$$

where $f_0 = F/N$ and η is a random (uncorrelated) noise with a Gaussian distribution

$$p(\eta) = \frac{1}{\sqrt{2\pi kT}} \exp\left(-\frac{\eta^2}{2kT}\right) \tag{2}$$

of zero mean and variance kT, following precisely the model and notations introduced in Ref. [17]. The cumulative distribution of η is called $P(\eta) = \int_\eta^\infty p(x)dx$.

The probability that one fiber survives after time t is

$$p_1(t) = [1 - P(1 - f_0)]^t. \tag{3}$$

The probability that the entire bundle survives after time t is

$$p_N(t) = [1 - P(1 - f_0)]^{Nt}. \tag{4}$$

Thus the distribution of the first failure time is an exponential distribution. The average failure time is

$$\langle \tau_1 \rangle = \frac{-1}{N \ln[1 - P(1 - f_0)]}. \tag{5}$$

Once the first fiber is broken, one faces a similar problem with a smaller fiber bundle, and a larger load per fiber. Thus after $i-1$ broken fibers, the next failure is again an exponential distribution of average time $\langle \tau_i \rangle$

$$\langle \tau_i \rangle = \frac{-1}{(N-i)\ln\{1 - P[1 - N f_0/(N-i)]\}}. \tag{6}$$

Therefore the total failure time for the bundle is

$$\langle \tau_f \rangle = \sum_{i=1}^{N} \frac{-1}{(N-i)\ln\{1 - P[1 - N f_0/(N-i)]\}}. \tag{7}$$

For large N, we can turn this sum into a continuous integral

$$\langle \tau_f \rangle = N^{-1} \int_0^N \frac{-N}{(N-x)\ln\{1 - P[1 - N f_0/(N-x)]\}} dx$$
$$= \int_{f_0}^{\infty} \frac{-1}{\ln[1 - P(1-y)]} \frac{dy}{y}. \tag{8}$$

It is important to note that the size of the fiber bundle N disappears in this expression. The above integral is an exact result which involves no approximation. In fact the above analysis Eqs. (3)–(8) is only a specific example of a general result obtained first by Coleman. [10] The quantity $P(1-y)$ was termed ''breakdown rule'' in this reference, and a number of properties have been illustrated in specific examples, such as exponential or a power-law function [19,18] for this breakdown rule. In the following, we will illustrate our discussion with simple numerical simulations which essentially consist in integrating the above equation numerically, using for the ''breakdown rule'' the specific ''error function'' which results from the definition of the model in Ref. [17].

It is impossible to arrive at a closed-form expression for the failure time for a Gaussian distribution of force fluctuations. However, we may use the fact that the sum is dominated by the time required for breaking the first fibers when (i) f_0 is much less than the maximum load $f_c = 1$ a fiber can sustain and (ii) the force fluctuations have a small amplitude $kT \ll 1$. Then P can be considered as much smaller than 1 in the above expression. To reach an analytical expression for the failure time, we thus expand the derivative of τ_f with respect to f_0 for small P, and integrate this expansion. The derivative of this time with respect to f_0 gives

$$\frac{\partial \langle \tau_f \rangle}{\partial f_0} = \frac{1}{f_0 \ln[1 - P(1 - f_0)]} \approx \frac{1}{f_0 P(1 - f_0)}. \tag{9}$$

For $(1-f_0)^2 \gg kT$, we may expand the error function to obtain

$$P(1 - f_0) = \frac{\sqrt{kT} \exp\left(-\frac{(1-f_0)^2}{2kT}\right)}{\sqrt{2\pi}(1 - f_0)} [1 + \mathcal{O}(kT)]. \tag{10}$$

Taking into account only the dominant terms in powers of kT, we obtain

$$\langle \tau_f \rangle \approx \frac{\sqrt{2\pi kT}}{f_0} \exp\left(\frac{(1-f_0)^2}{2kT}\right). \tag{11}$$

For $kT \ll (1-f_0)^2$, the above approximation gives an excellent approximation as shown below.

Guarino *et al.* [17] have considered the above problem through numerical simulations. Different properties of the model were investigated in order to draw a comparison with either experimental results published in Refs. [15,16,20] or theoretical suggestions by Pomeau. [14] Concerning the failure time under a constant load, Guarino *et al.* reported the following two key observations: (1) $\ln\langle \tau_f \rangle \propto f_0^{-2}$ [Fig. 2(a) of Ref. [17]] and (2) $\ln\langle \tau_f \rangle \propto 1/(kT)$ [Fig. 2(b) of Ref. [17]].

None of these two results is actually exact. The first result seems to be a fortuitous coincidence. Indeed, using the parameter range used in Ref. [17], the data in a $\ln\langle \tau_f \rangle$ vs $1/f_0^2$ graph shows only a limited curvature. Figure 1 proposes a graph similar to Fig. 2(a) of Ref. [17], and we indeed observe that the data can be reasonnably considered as straight over this range of variation, in agreement with the numerical study of the latter reference. However, as the range of forces is extended, the apparent linearity breaks down. We note on the figure that the approximation given in Eq. (11) shown as thin lines gives an excellent agreement with the direct computation of the integral Eq. (8).

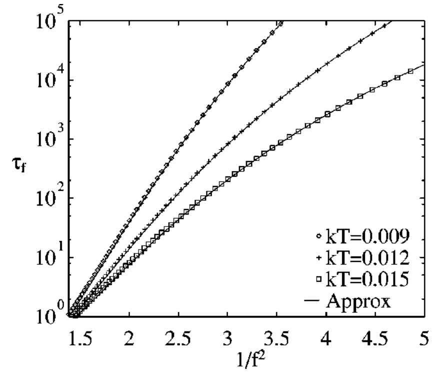

FIG. 1. Mean failure time (log scale) plotted as a function of $1/f^2$. The three curves correspond to $kT=0.009$, 0.012, and 0.015 from top to bottom. The thin curves show the approximate expression obtained in Eq. (11).

The second result proposed by Guarino *et al.*, [17] $\ln(\tau_f)\propto 1/(kT)$ differs from our result Eq. (11) only through a weak logarithmic correction. Figure 2 indeed shows that the data conforms to such a variation for low temperatures, and intermediate forces. Again we note that the integral Eq. (8) is well approximated by Eq. (11).

III. SCALING ARGUMENT

As the above argument does not provide much physical insight in the expression of the failure time, we present a simpler argument which reproduces the leading expression of τ_f. Let us consider the failure time $\langle\tau_1\rangle$ for the first fiber. Using the above hypothesis of a low temperature or small fluctuating part for the force $(1-f_0)^2 \gg kT$, we can write

$$\langle\tau_1\rangle \approx \frac{1}{NP(1-f_0)} \approx \frac{\sqrt{2\pi}(1-f_0)}{N\sqrt{kT}}\exp\left(\frac{(1-f_0)^2}{2kT}\right). \tag{12}$$

This time is decreasing as the number of the broken fiber increases. The total failure of the bundle is reached after a time which can be roughly estimated as $\langle\tau_1\rangle$ times the number of fibers necessary to reduce significantly the breaking time. The initiation stage can be estimated by noting that the most rapidly varying term is provided by the exponential, and thus the argument of the exponential drops for a change in force supported by the fibers of order

$$\Delta f \propto \frac{kT}{(1-f_0)}. \tag{13}$$

This change occurs after n^* fiber have failed where $f_0(n^*/N)=\Delta f$, hence

$$n^* \propto \frac{NkT}{(1-f_0)f_0} \tag{14}$$

Therefore, the failure time τ_f is finally estimated as $n^*\tau_1$, or

$$\tau_f \propto \sqrt{\frac{kT}{f_0}}\exp\left(\frac{(1-f_0)^2}{2kT}\right) \tag{15}$$

in agreement with the approximate expression Eq. (11).

IV. FAILURE TIME DISTRIBUTION

We have studied the mean value of the failure time distribution. Let us now consider the statistical distribution of these failure times. We have seen that τ_f was the sum of exponentially distributed times τ_i each independent, but with a different characteristic time (and thus average, standard deviations, etc. are all affected by this scaling). At fixed force f_0 and temperature, as the system size goes to infinity, the central limit theorem holds, and thus the global failure time is Gaussian distributed. The variance of the distribution is also simply additive, and thus its value, σ^2, for the global failure time is

$$\begin{aligned}\sigma^2 &= \langle(\tau_f - \langle\tau_f\rangle)^2\rangle \\ &= N^{-2}\int_0^N \frac{-N^2}{(N-x)^2 \ln[1-P[1-Nf_0/(N-x)]]^2}dx \\ &= \frac{1}{Nf_0}\int_{f_0}^{\infty}\frac{1}{\ln[1-P(1-z)]^2}dz.\end{aligned} \tag{16}$$

We again resort to the low temperature hypothesis, and apply a similar computation as for the previous section. We obtain

$$\sigma^2 \approx n^*\tau_1^2 \approx \frac{2\pi(1-f_0)}{Nf_0}\exp\left(\frac{(1-f_0)^2}{kT}\right). \tag{17}$$

Thus as the size of the fiber bundle increases, the width of the distribution of failure times is expected to become more narrow, scaling as $1/\sqrt{N}$. The relative width of this distribution, normalized by the mean failure time $\sigma^2/\tau_f^2 \propto 1/n^*$, also scales as $1/kT$, giving a broader distribution for low temperatures. Let us note that this result has been obtained by Coleman [10,11] and generalized by Phoenix [12,13] under much more general assumptions.

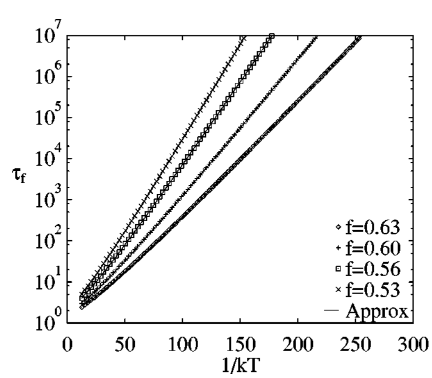

FIG. 2. Mean failure time (log scale) plotted as a function of $1/kT$. The force f in the four curves correspond to 0.63, 0.60, 0.56, and 0.53 as indicated in the caption. The continuous thin curves show the approximate formula Eq. (11) and the symbols correspond to the numerical integration of Eq. (8).

V. DISORDERED CASE

Now we consider a statistical distribution of breaking force threshold $f_c^{(j)}$ for each fiber j, supposed to distributed as a Gaussian of average 1, and variance $k\Theta$, where following

Guarino *et al.* [17] we introduce an equivalent temperature Θ to measure this time-independent breaking force variance. The probability that fiber j survives after time t is

$$s(t)=[1-P(f_c^{(j)}-f_0)]^t. \tag{18}$$

The probability that the entire bundle does not experience any failure after time t is

$$s(t)=\prod_{j=1}^{N}[1-P(f_c^{(j)}-f_0)]^t. \tag{19}$$

For a large number of fibers N we may expand

$$s(t)=\exp(-t/\tau_1) \tag{20}$$

with

$$1/\tau_1=-N\int_{-\infty}^{\infty}\ln[1-P(1+x-f_0)]g(x)dx, \tag{21}$$

where g is the Gaussian distribution of zero mean and variance $k\Theta$. The interesting regime is when both temperatures T and Θ are low, i.e., $kT\ll(1-f_0)^2$ and $k\Theta\ll(1-f_0)^2$. Then one may expand the (small) probability P, and obtain

$$1/\tau_1=N\sqrt{\frac{T}{2\pi\sqrt{\Theta}}}\exp\left(-\frac{(1-f_0)^2}{2kT}\right)\int_{-\infty}^{\infty}\frac{1}{(1+x-f_0)}\times[1-\mathcal{O}(kT)]\exp\left(-\frac{(T+\Theta)x^2}{2kT\Theta}\right)\exp\left(-\frac{x(1-f_0)}{kT}\right)dx. \tag{22}$$

Simple algebra retaining only the leading term (low temperatures) in the expansion gives for $(1-f_0)^2\gg k\Theta(T+\Theta)/T$

$$\tau_1=\frac{\sqrt{2\pi}}{N}\frac{(1-f_0)}{\sqrt{k(T+\Theta)}}\exp\left(\frac{(1-f_0)^2}{2k(T+\Theta)}\right). \tag{23}$$

The remarkable property of this expression is that it can be compared to the previous result obtained for a homogeneous system, Eq. (12), with, however, a different temperature $T_{\rm eff}$ such that

$$T_{\rm eff}=T+\Theta. \tag{24}$$

This conclusion is similar to the one proposed by Guarino *et al.* [17] on the basis of their numerical simulations, namely, that the ''quenched'' (time-independent) heterogeneities of the medium modifies the effective temperature. Moreover it appears as being simply additive (for low temperatures). It is important to note that fitting the experimental fracture data for various systems [20,16] revealed a good agreement with a theoretical expectation expression for thermally activated failure only if a temperature much larger than the actual one of the experiment was used. One possibility is that indeed quenched disorder comes into play through an effective temperature such as the one obtained above. However, one should also consider the possibility that the actual form of the fitting expression is not quite relevant for the experiment. A systematic study system by system should be performed to resolve those possibilities.

The above computation concerns the first fiber failure. However, we are interested in the failure of the entire fiber bundle, which requires the breaking of a large number of fibers, previously estimated to be of order n^* [Eq. (14)]. It is tempting to extrapolate the previous scaling argument. However, as more and more fibers fails, the tail of the distribution of force threshold is ''consumed'' in the process, and thus, the ''temperature'' Θ associated with the distribution of breaking threshold should decrease.

However, in the low-temperature regime $kT\ll 1$, we have seen that the failure time was dictated by the rupture of a small proportion of fibers n^*/N proportional to kT. In the presence of disorder, this proportion may be slightly enhanced (with an upper bound obtained with $T_{\rm eff}$ instead of T in the expression of n^*). Nevertheless as n^*/N goes to zero, the above estimate is expected to give a proper account of the role of an additional quenched noise.

VI. FRACTURE PRECURSORS

Another extremely interesting point was raised in the numerical study of Guarino *et al.*, [17] namely, the scaling of precursors to fracture. This point is of fundamental and practical interest. It has been addressed in the past for the fiber bundle case (without any thermal noise) through the study of avalanche statistics. This problem has been analytically resolved by Hemmer and Hansen. [4] Let us here recall briefly the results obtained by Guarino *et al.*

In an elementary fiber failure, the elastic energy stored in the nth fiber which breaks is

$$\epsilon=(1/2)\frac{F^2}{(N-n)^2}=(1/2)\frac{f_0^2}{(1-n/N)^2} \tag{25}$$

since the stiffness of each fiber is unity. In Ref. [17] the distinction is made between individual fiber failure and events when several fibers break simultaneously. The notion of simultaneity is related to the short range correlation in the thermal noise, and thus we will ignore this and only discuss events as if they consisted in a single fiber. In fact events will consist of many fibers only in the very end of the process [when $f_0/(1-n/N)$ is of order $f_c=1$], where essentially one big event is expected. The latter in all cases will have a singular scaling as compared to the previous ones. It was found that the statistics of ϵ display a power-law distribution, considering all events up to fracture.

Let us show that this can be proved in this model. We will here resort to the homogeneous case for simplicity. The total number of events carrying an energy less than ϵ_0 is obtained from inverting Eq. (25) since ϵ is a monotonous function of n

$$\mathcal{N}(\epsilon<\epsilon_0)=N-\frac{F}{\sqrt{2\epsilon_0}} \tag{26}$$

valid for $\epsilon_0<1/2$ where the load per fiber reaches the threshold [with $n=n_c=N(1-f_0/f_c)$]. Taking the derivative of this cumulative distribution with respect to ϵ_0 gives the distribution $\mathcal{N}'(\epsilon)$

$$\mathcal{N}'(\epsilon)=\frac{F}{(2\epsilon)^{3/2}}. \tag{27}$$

The normalization is obtained from the number of such events, i.e., n_c, so that the normalized distribution is

$$\mathcal{N}'(\epsilon)=\frac{f_0}{(1-f_0)(2\epsilon)^{3/2}}. \tag{28}$$

Thus the distribution of ϵ is power-law distributed with an exponent 3/2. Such an exponent is consistent with the data (although not with the proposed fit) obtained by Guarino *et al.* and shown in Fig. 1(a) of Ref. [17].

Finally, these authors also studied the total energy released as a function of time, or more precisely of rescaled time to failure $\tilde{t}=(\tau_f-t)/\tau_f$. In fact the total energy released up to the failure time is finite, and can be computed from the force-displacement history of the fiber bundle. Stopping right before unstable fracture, the total energy $E=\Sigma\epsilon_n$ amounts to

$$E_{f1}=N\frac{f_0(1-f_0)}{2} \tag{29}$$

while the unstable final event will release an energy $E_{f2}=Nf_0/2$. Ignoring the latter, it is easy to obtain the total energy which has been released after $n\equiv xN$ fibers have been broken:

$$E(x)=(N/2)\frac{f_0^2x}{1-x}. \tag{30}$$

Similarly, one can easily compute the time to failure from any intermediate stage, redefining the bundle size $N'=(1-x)N$, and the force $f_0'=f_0/(1-x)$. We have

$$\tau(x)=\frac{\sqrt{2\pi kT}(1-x)}{f_0}\exp\left(\frac{(1-x-f_0)^2}{2kT(1-x)^2}\right). \tag{31}$$

So that the reduced time is

$$\tilde{t}=\frac{\tau_f-t}{\tau_f}=\frac{\tau(x)}{\tau(0)}=(1-x)\exp\left(\frac{(1-x-f_0)^2}{2kT(1-x)^2}-\frac{(1-f_0)^2}{2kT}\right). \tag{32}$$

Note, however, that this expression is inadequate when x becomes very close to x_f, since $\tilde{t}$ does not reach 0 exactly at this point. However, we recall that this expression was obtained in the limit $kT\ll((1-f_0)$ and thus $\tilde{t}$ is very small for $x=x_f$. The domain of validity of this law is, however, expected to hold for most of the controlled failure stage. We will investigate numerically this statement in the following.

The proportion of fibers broken at the onset of instability is $x_f=1-f_0$, so that the rescaled time can be expressed as

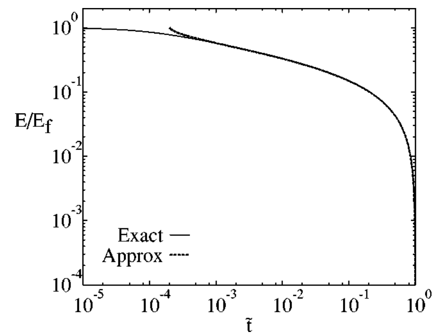

FIG. 3. Log-log plot of the total energy dissipated up to (reduced) time $\tilde{t}$. The thin line corresponds to the direct integration of the data, whereas the thick dotted line corresponds to the approximate expression from the parametric form Eqs. (33) and (30). In this example $f_0=0.6$ and $kT=0.01$.

$$\tilde{t}=(1-x)\exp\left(\frac{(x_f-x)^2}{2kT(1-x)^2}-\frac{x_f^2}{2kT}\right). \tag{33}$$

Finally the total energy dissipated as a function of time is obtained from the elimination of x between Eqs. (30) and (33).

Guarino *et al.* [17] proposed that $E(x)$ diverges as a power law of $\tilde{t}$. We have shown that $E(x)$ is bounded from above and thus such a law cannot hold. We show in Fig. 3 a plot similar to Fig. 1(b) of Ref. [17]. In this figure, we used both the exact expression for the time, and the above Eq. (33) to check the validity of the approximation. We do observe a good fit for most of the reduced time parameter. The power law reported by Guarino can indeed be fitted to the data, with a very small exponent. This power-law does not hold over an increasing range of parameters, even if the system size diverges, and thus the parallel with a critical-like behavior appears to be somewhat fortituous.

Since we know that $E(\tilde{t})$ approaches a constant for $\tilde{t}\to 0$, we may rather study

$$e\equiv\frac{[E_{f1}-E(\tilde{t})]}{E_{f1}} \tag{34}$$

as a function of $\tilde{t}$. Figure 4 shows the graph of this function for the same parameters as Fig. 3. We observe that the approximate expression given above (shown as a bold dotted curve) reproduces most of the behavior apart from the immediate vicinity of the global failure. But the latter may be very sensitive to the detailed numerical implementation of the model.

In order to get some more insight in the analytical expression of the energy dissipated, we resort to the parametric form and propose to analyze it. Focusing on the neighborhood of global failure, we substitute x_f to x in all nonsingular expressions, and we ignore logarithmic corrections to obtain

$$e\approx\sqrt{1+\frac{2kT}{(1-f_0)^2}\ln(\tilde{t})}. \tag{35}$$

Again we stress that this expression is expected to hold for moderate $\tilde{t}$, and not vanishingly small values. Indeed, this

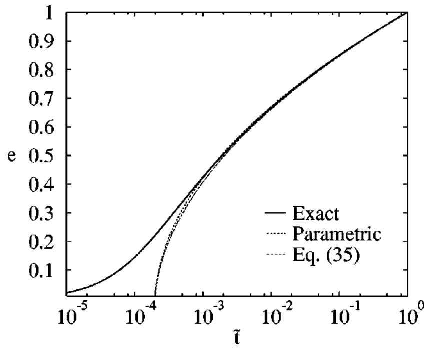

FIG. 4. Log-log plot of $e=[E_{f1}-E(\tilde{t})]/E_{f1}$, the reduced energy which will be dissipated from $\tilde{t}$ to the global failure, as a function of the (reduced) time $\tilde{t}$. The bold continuous curve corresponds to the direct integration of the data, whereas the thick dotted line is the approximate expression from the parametric form, Eqs. (33) and (30) and the thin dashed curve is the simplified expression Eq. (35). In this example $f_0=0.6$ and $kT=0.01$.

simpler form fits the data very accurately for $\tilde{t}>0.001$. Thus, we obtain that the total energy dissipation approaches a constant for long times, with a very slow increase, $\sqrt{-\ln \tilde{t}}$, in contrast with the early suggestion of Ref. [17]. Moreover, from the data shown in Ref. [17], the asymptotic value predicted here $E_{f1}+E_{f2}$ (since no distinction was made for the stable and unstable failure) is very precisely obtained.

VII. CONCLUSION

We have studied the failure time distribution of thermally activated breakdown in a simple fiber bundle model as proposed by Guarino *et al.* [17] Analytical expressions have been derived in the limit of small temperature for the mean failure time as well as its entire statistical distribution. The quenched disorder case has been shown to display a similar behavior as the homogeneous case, with the introduction of an effective temperature accounting for the initial disorder.

Finally, we have addressed the question of fracture precursors in this model, and we have proven the power-law distribution of energy release events. The total energy dissipation as a function of time to failure has also been considered, and an approximate law was proposed describing the very slow increase toward a finite value, in contrast to earlier suggestions of an algebraic divergence.

Application of this model to the mechanical behavior of fibrous materials is a very important issue. Albeit the description used here is very schematic, delayed damaged may occur for, e.g., glass fibers under stress, and it would be of interest to see how much such an approach can contact with experiments. Ongoing work on glass wool [22] shows that this material is a natural candidate for such a comparison.

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